DOUBLED FIELD APPROACH TO YANG - MILLS REQUIRES NON-LOCALITY

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Abstract. Doubling a Yang-Mills field we apply the pattern which has been found to construct a "duality-symmetric" gravity with matter to the "duality-symmetric" Yang - Mills theory in five space-time dimensions. Constructing the action we conclude that dualizing a non-abelian theory requires non-locality. We analyze the symmetries of the theory and equations of motion. Extension to the supersymmetric theory is also demonstrated.

Some of the fields of Superstring/M-theory spectrum are of special class which are called chiral p-forms, or chiral bosons. These fields play an important rôle in establishing various dualities between different sectors of M-theory, but dealing with them beyond the mass-shell, i.e. at the level of the effective Lagrangians, is not a simple task. There are several methods (see [1] for a review and for the comprehensive list of Refs.) which were proposed to describe theories with self-dual or duality-symmetric fields. All of them could be split into the following major sets. The first one [2]-[6] contains the approaches that are duality invariant but are not manifestly Lorentz invariant. Introducing auxiliary fields is not required, but coupling to other fields, especially to gravity, may cause problems with establishing the consistency of such a coupling. The second set [7]-[12] is dealt with auxiliary fields whose inclusion restores the Lorentz covariance. The number of these auxiliary fields may vary from one to infinity.

Among the approaches with auxiliary fields the formalism proposed by Pasti, Sorokin and Tonin [12] takes a special place. It is manifestly Lorentz covariant and is minimal in a sense of having the only auxiliary field entering the action in a non-polynomial way. Successful applying the PST approach to the construction of different field theories of chiral p-forms, super-p-branes with worldvolume chiral fields, and of different sub-sectors of supergravities has demonstrated the advantages of this approach and its compatibility with supersymmetry (cf. [1] and Refs. therein). However, a gap for applying the PST formalism is a Yang-Mills theory.

It is worth noting that the problem of dualising a non-abelian gauge theory has been intensively studied in literature. Getting rid of self-interactions it is straightforward to apply the machinery of dualization to the case. But the attempts to go beyond the free theory have faced the troubles. The latter

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can be summarized in the "no-go" theorems [3], [13] which forbid a trivial generalization of the well-known electric-magnetic duality of Maxwell theory. The point is that the Poincaré lemma does not directly generalize when dealing with a YM covariant derivative that in its turn prevents the straightforward applying the PST formalism to formulate a duality-symmetric YM theory [14], [15].

Searching for a dualization of a non-abelian theory becomes now important from the point of view of pushing forward the doubled field approach of [16], [17] to the non-maximal supergravities. On the supergravity side much has been done in the dualization program for maximal supergravities in diverse dimensions [16], [17] that can be obtained from D=11 or D=10 IIA/B supergravities by toroidal dimensional reduction. There doubling the fields of the gauge sector of supergravities it has been demonstrated that the original equations of motion of a theory admit the representation in terms of the Bianchi identities for the dual fields, and moreover, the dynamical content of a theory is encoded into the so-called twisted self-duality condition relating the original and the dual field strengths. The way to go beyond the mass-shell by lifting the approach of [17] onto the level of the proper action was proposed for D=11 and D=10 type IIA supergravities in [18], [19]. However, the doubled field approach can not be directly applied for the non-maximal supergravities in D=10 and for the low-dimensional gauged supergravities where the non-abelian fields become a part of the supergravity multiplet.

Therefore, to realize the aforementioned program we are forced to figure out a way to deal with non-abelian fields. One of the ways is to find a generalization of the Hodge star notion to a non-abelian case. Such a generalization has been proposed in [20] and requires essentially non-local consideration since it is based on a loop space formulation of a gauge theory. We are aimed to reach the same conclusion on the non-local character of a dualization of a non-abelian theory being on the ground of standard approach to the Yang-Mills theory. Since the YM theory possesses the same as the gravity theory feature of having a self-interaction we will use this fact to establish the properties of the doubled field approach to the (S)YM theory which will help us in pushing forward the same approach to the (super)gravity case. And since we will mostly interested in a dualization of D=11 (super)gravity, in a spirit of recent studying a hidden symmetry group of M-theory [25]-[41], we will exploit the tight relation between five-dimensional simple supergravity and D=11 supergravity [21] to study the YM theory in D=5.

As a first step in recovering the doubled field action for the YM theory we have to find, without an appeal to a method of constructing such an action, a convenient representation of the YM equation of motion in a way that allows us to present the latter as the Bianchi identity for a dual field. It turns out to be convenient to write down the YM equation of motion in a form which is very similar to the dynamics of Maxwell theory with an electric-type source. Such a representation suggests a way of extracting the dual field after that we can apply the machinery of the PST approach to construct the action from which the duality relations between the YM field and its dual partner will follow as equations of motion.

To do so let us get started with the following action for a Yang-Mills theory in D=5

$$S_{YM} = -\frac{1}{2}\mathrm{Tr}\int_{\mathcal{M}^5} F^{(2)} * F^{(2)}, \tag{1}$$

where $F^{(2)} \equiv DA^{(1)} = dA^{(1)} - \frac{1}{2}ig[A^{(1)}, A^{(1)}]$ is the field strength in the adjoint representation of a semi-simple non-abelian group, g is a coupling constant, and the wedge product between forms has to be assumed. The action S_{YM} and the gauge fields equation of motion

$$D * F^{(2)} = 0 \tag{2}$$

are invariant under the local non-abelian gauge transformations

$$\delta A^{(1)} = \frac{1}{g}D\alpha^{(0)} \equiv \frac{1}{g}d\alpha^{(0)} + i[A^{(1)}, \alpha^{(0)}]. \tag{3}$$

To apply the Poincaré lemma let us present the equation of motion (2) in a slightly different form extracting the part with usual, non-covariant, derivative, and separating the free part from that of describing the self-interaction of a YM field. Taking into account the definition of the YM field strength we get

$$d(*dA^{(1)}) = *J^{(1)} \tag{4}$$

with

$$*J^{(1)} = ig[A^{(1)}, *F^{(2)}] + d * \left(\frac{1}{2}ig[A^{(1)}, A^{(1)}]\right). \tag{5}$$

Since $d^2 = 0$, in the trivial topology setting one could notice

$$*J^{(1)} = d * G^{(2)}, \tag{6}$$

where $G^{(2)}$ is a function of the YM potentials $A^{(1)}$ and their derivatives and as we will see in what follows is the source of non-locality since formally we can resolve (6) through the non-local expression

$$*G^{(2)} = d^{-1} * J^{(1)}. \tag{7}$$

Here we have introduced the inverse to d operator whose action can be understood as follows. Let us introduce a "Green" function to the equation

$$dh(x) = \delta^5(x), \tag{8}$$

with the Dirac delta-function on the r.h.s. Then we define the action of d^{-1} on an arbitrary form at a space-time point x as

$$d^{-1}(x)\omega^{(p)}(x) = (-)^p \int d^5y\, h(x-y)\ \omega^{(p)}(y). \tag{9}$$

To make a sense the latter expression should only deal with the "Green" functions that act on a causality-related space-time region.

The dual to the one-form $J^{(1)}$ is a conserved "current" which is the YM analog of the gravity Landau-Lifshitz pseudo-tensor, and eq. (4) is an analog of the equation of motion of Maxwell field with an electric-type current. Note that the "current" entering the r.h.s. of (4) is not gauge invariant, but since the l.h.s. of the same equation is not gauge invariant too, the latter compensates the former leaving eq. (4) to be invariant under the local gauge transformations (3).

Having the representation (4) we can double the YM field with its "dual" partner and write down this equation of motion as the Bianchi identity for the YM "dual"

$$dB^{(2)} = *(dA^{(1)} - G^{(2)}), \tag{10}$$

or equivalently

$$\mathcal{F}^{(3)} = 0, \qquad \mathcal{F}^{(3)} = dB^{(2)} - *(dA^{(1)} - G^{(2)}). \tag{11}$$

Indeed, applying the operator d to (10) or (11) leads to the YM equation of motion (2)

$$d\mathcal{F}^{(3)} = D * F^{(2)} = 0. \tag{12}$$

Therefore, an equivalent way of a description of a YM theory is to find the action from which eq. (11) will follow as an equation of motion.

However, the action we shall construct should be gauge invariant as well as equations of motion which will follow from that action. Therefore, we get to inspect the gauge invariance more closer. To this end let us recall that the action of the local gauge transformations (3) on the YM field strength results in the rotation of the latter in a group space, i.e. $\delta F^{(2)} = -i[\alpha^{(0)}, F^{(2)}]$. To find the similar transformation law for the $\mathcal{F}^{(3)}$ it is convenient to present (4) as

$$\begin{aligned} &d * F^{(2)} = * \tilde{J}^{(1)}, \\ &\qquad * \tilde{J}^{(1)} = ig[A^{(1)}, *F^{(2)}] = d * \tilde{G}^{(2)}. \end{aligned} \tag{13}$$

Then, using the $F^{(2)}$ gauge transformation law one can derive from (13) the transformation of $*\tilde{G}^{(2)}$

$$\delta * \tilde{G}^{(2)} = -i[\alpha^{(0)}, \mathcal{F}^{(3)} + *F^{(2)}] - d^{-1}\left(i[d\alpha^{(0)}, \mathcal{F}^{(3)}]\right), \tag{14}$$

which is the non-local gauge transformation in view of the non-local character of this quantity.

To require $\delta\mathcal{F}^{(3)} = -i[\alpha^{(0)}, \mathcal{F}^{(3)}]$ one has to assign the following non-local non-abelian gauge transformation to the $B^{(2)}$ field

$$\delta B^{(2)} = d\alpha^{(1)} + d^{-2}\left(i[d\alpha^{(0)}, \mathcal{F}^{(3)}]\right). \tag{15}$$

What concerns to the gauge invariance of the equations of motion, we should emphasize that the standard YM equation of motion (2) is only on-shell invariant under the action of (3) since $\delta(D * F^{(2)}) = -i[\alpha^{(0)}, D * F^{(2)}]$. The same concerns to the $d\mathcal{F}^{(3)} = 0$ since this expression is gauge invariant only on the shell of the duality relation $\mathcal{F}^{(3)} = 0$.

Therefore, our attempt to stay on the ground of applying usual Poincaré lemma to the non-abelian case has faced the necessity of dealing with non-locality due to the "source"-like terms which appear in the non-abelian gauge field equation of motion after fitting the later for the application of the Poincaré lemma. It is easy to see that this non-localities disappear in the zero gauge coupling constant limit $g \to 0$ when $G^{(2)} \to 0$, $\mathcal{F}^{(3)} \to dB - *dA$, $\delta A = d\tilde{\alpha}^{(0)}$, $\delta B^{(2)} = d\alpha^{(1)}$, and therefore we are effectively dealing with $\mathcal{N}$ copies of the abelian duality-symmetric fields where $\mathcal{N}$ is the dimension of the non-abelian group. However, it does not contradict with the "no-go"

theorem of [15] since the extention of a system of $\mathcal{N}$ copies of free duality-symmetric abelian fields to a non-abelian system comes through introducing the non-local quantities. The other feature of the construction that has to be noticed consists in a non-equivalence of the original YM field and its "dual" partner since the non-abelian extension of the latter is formed by the part containing the self-interaction of the former. Indeed, the equation of motion for the dual field is

$$d(*dB^{(2)}) = -dG^{(2)}, \tag{16}$$

that follows from the duality relation $*\mathcal{F}^{(3)} = 0$. However, one can essentially simplify this equation with taking into account the Hodge identity

$$d\triangle^{-1}\delta + \delta\triangle^{-1}d = 1, \tag{17}$$

where δ is the co-derivative and $\triangle^{-1}$ is the inverse to the Laplacian $\triangle = d\delta + \delta d$ operator. Using the Hodge identity one can present $*\mathcal{F}^{(3)} = 0$ as

$$*dB^{(2)} = F^{(2)} - *\triangle^{-1}\delta * \tilde{J}^{(1)}, \tag{18}$$

and since the last term on the r.h.s. of the latter equation is a closed form, the equation of motion of $B^{(2)}$ is

$$d(*dB^{(2)}) = -\frac{1}{2}igd\left([A^{(1)}, A^{(1)}]\right). \tag{19}$$

Therefore, there is not a symmetry similar to the symmetry under duality rotations in Maxwell theory that is closely related to the "no-go" theorem of [3].

Let us now turn to the construction of the action from which the duality relation $\mathcal{F}^{(3)}$ will follow as an equation of motion. Taking into account an analogy with the gravity case considered in [22] it is quite naturally to guess the following term

$$S_{PST} = \frac{1}{2}\mathrm{Tr}\int_{\mathcal{M}^5} v\ \mathcal{F}^{(3)}\ i_v\mathcal{F}^{(2)}, \tag{20}$$

as the main candidate that has to be added to the action (1). Here the one-form v is constructed out of the PST scalar field $a(x)$ ensuring the covariance of the model

$$v = \frac{da(x)}{\sqrt{-(\partial a)^2}}, \tag{21}$$

$\mathcal{F}^{(3)}$ has appeared in (11), and

$$\mathcal{F}^{(2)} = dA^{(1)} - *(dB^{(2)} + *G^{(2)}), \quad \mathcal{F}^{(3)} = - * \mathcal{F}^{(2)}. \tag{22}$$

It is clear from the previous discussion that the generalized field strengths (11) and (22) are the covariant under the gauge transformations objects, although the quantities entering them are not covariant and the action (20) is invariant under the gauge transformations (3), (14), (15) which leave the PST scalar intact.

To prove the relevance of the proposed term, let us consider a general variation of (20). The standard manipulations (see [18] for details) result in

$$\begin{aligned}\delta\mathcal{L}_{PST} = \mathrm{Tr}\left(\delta B^{(2)} + \frac{\delta a}{\sqrt{-(\partial a)^2}} i_v \mathcal{F}^{(3)}\right)\ d(v\ i_v \mathcal{F}^{(2)}) \\ - \mathrm{Tr}\left(\delta A^{(1)} + \frac{\delta a}{\sqrt{-(\partial a)^2}} i_v \mathcal{F}^{(2)}\right)\ d(v\ i_v \mathcal{F}^{(3)}) \\ - \mathrm{Tr}\ \delta(*G^{(2)})\ v\ i_v\mathcal{F}^{(2)} - \mathrm{Tr}\ \delta A^{(1)}\ d\mathcal{F}^{(3)},\end{aligned} \tag{23}$$

where we have omitted the total derivative term.

The last term of (23) is precisely the term whose contribution is cancelled against the variation of S_{YM}. Therefore, the complete action $S = S_{YM} + S_{PST}$ is invariant under the non-abelian gauge transformations and the following two sets of special symmetry [12]

$$\begin{aligned}\delta a(x) = 0, \qquad & \delta A^{(1)} = da\ \varphi^{(0)}, \\ & (d\delta B^{(2)} + \delta * G^{(2)}) = da\ d\varphi^{(1)} \Longrightarrow \\ & \delta B^{(2)} = da\ \varphi^{(1)} - d^{-1}\delta(*G^{(2)}),\end{aligned} \tag{24}$$

$$\begin{aligned}\delta a(x) = \Phi(x), \quad \delta A^{(1)} &= -\frac{\delta a}{\sqrt{-(\partial a)^2}} i_v\mathcal{F}^{(2)}, \\ \delta B^{(2)} &= -\frac{\delta a}{\sqrt{-(\partial a)^2}} i_v\mathcal{F}^{(3)} - d^{-1}\delta(*G^{(2)}).\end{aligned} \tag{25}$$

Let us now discuss how these special symmetries do the job. The equations of motion of $B^{(2)}$ and $A^{(1)}$ that follow from the action $S = S_{YM} + S_{PST}$ are

$$d(v\ i_v\mathcal{F}^{(2)}) = 0, \tag{26}$$

$$d(v\ i_v\mathcal{F}^{(3)}) + \frac{\mathrm{Tr}(v\ i_v\mathcal{F}^{(2)}\ \delta * G^{(2)})}{\delta A^{(1)}} = 0, \tag{27}$$

where we have used that $*G^{(2)}$ is a function of the YM potentials $A^{(1)}$.

The general solution to the equation of motion (26) is [12]

$$v\ i_v\mathcal{F}^{(2)} = da\ d\xi^{(0)}. \tag{28}$$

Using the symmetry (24) with $\varphi^{(0)} = \xi^{(0)}$ one can obtain from (28)

$$i_v\mathcal{F}^{(2)} = 0 \quad \rightsquigarrow \quad \mathcal{F}^{(2)} = 0. \tag{29}$$

Taking the latter into account and using the same trick one can obtain from (27)

$$i_v\mathcal{F}^{(3)} = 0 \quad \rightsquigarrow \quad \mathcal{F}^{(3)} = 0. \tag{30}$$

It becomes clear that equation of motion of the PST scalar $a(x)$

$$\mathrm{Tr}\left(i_v\mathcal{F}^{(3)}\ d(v\ i_v\mathcal{F}^{(2)}) - i_v\mathcal{F}^{(2)}\ d(v\ i_v\mathcal{F}^{(3)})\right) = 0 \tag{31}$$

does not contain a new dynamical information and is satisfied identically as a consequence of the equations of motion (26), (27). Indeed, eq. (31) is the

Noether identity which is a reflection of a local symmetry which is nothing but the symmetry under (25).

Therefore, we have proved that the action $S = S_{YM} + S_{PST}$ is the one we are looking for. The action possesses the special symmetries (24), (25) which have to be used to derive from equations of motion the duality relations between the YM field and its dual partner and to establish the auxiliary nature of the PST scalar field. Owing to the symmetry (25) the PST scalar field does not spoil the original content of a theory and is the pure auxiliary field.

To extend this construction to the supersymmetric case recall that the supersymmetric counterpart of the D=5 YM theory is described by

$$S_{SYM} = -\frac{1}{2}\ \mathrm{Tr} \int_{\mathcal{M}^5} \left(F^{(2)} * F^{(2)} + i\bar{\lambda}\Gamma^a D\lambda \Sigma_a + \dots \right), \tag{32}$$

where the four-form Σ_a is defined by

$$\Sigma_a = \frac{1}{4!}\epsilon_{abcde} E^b E^c E^d E^e \tag{33}$$

with the vielbeins E^a, and $D\lambda$ is the covariant derivative of the gaugino field. Note that we have kept only the terms essential for the consideration in what follows neglecting the scalars and auxiliary fields which cast the off-shell N=2 Yang-Mills supermultiplet in D=5.

The action (32) is in particular invariant under the following global supersymmetry transformations

$$\delta_\epsilon A^{(1)} = -\frac{i}{2}\bar{\epsilon}\Gamma^{(1)}\lambda, \qquad \delta_\epsilon \lambda = \frac{1}{2} * (*F^{(2)}\ \Gamma^{(2)})\epsilon, \tag{34}$$

where we have used the following notation for gamma-matrices

$$\Gamma^{(n)} = \frac{1}{n!} E^{a_n}\ \dots\ E^{a_1}\Gamma_{a_1 \dots a_n}. \tag{35}$$

To find the appropriate supersymmetry transformations for the doubled field version of the super-Yang-Mills theory it is convenient to present the PST part of the action as

$$S_{PST} = -\frac{1}{2}\ \mathrm{Tr} \int_{\mathcal{M}^5} i_v \mathcal{F}^{(2)} * i_v \mathcal{F}^{(2)} \tag{36}$$

with

$$\mathcal{F}^{(2)} = F^{(2)} - *(dB^{(2)} + *\mathcal{G}^{(2)}), \tag{37}$$

where $\mathcal{G}^{(2)}$ is the extention of $\tilde{G}^{(2)}$ with a non-local term coming from the fermionic current.

Then it is easy to verify that the action $S = S_{SYM} + S_{PST}$ is invariant under the following global supersymmetry transformations

$$\begin{aligned} \delta_\epsilon a = 0, \qquad \delta_\epsilon A^{(1)} &= -\frac{i}{2}\bar{\epsilon}\Gamma^{(1)}\lambda, \\ \delta_\epsilon \lambda &= \frac{1}{2} * \left(*[F^{(2)} + v\ i_v \mathcal{F}^{(2)}]\ \Gamma^{(2)} \right)\epsilon. \end{aligned} \tag{38}$$

The supersymmetry transformation of the dual to the YM field can be recovered from the requirement

$$\delta_\epsilon \left(*dB^{(2)} + \mathcal{G}^{(2)} \right) = 0. \tag{39}$$

Hence, the proposed extension of the PST technique to a non-abelian case is compatible with supersymmetry but requires the non-local terms in the supersymmetry transformation of the dual field.

To summarize, we have presented the Yang-Mills equation of motion in the form which is very likely to that of Maxwell theory with an electric-type current. The YM "current" form so obtained encodes the self-interaction between the Yang-Mills fields, does not possesses the local gauge invariance and is a closed form. The latter allows one to present the "current" form as a curl of a "current potential" and therefore to rewrite the second order YM equation of motion as the first order Bianchi identity for the dual field. Since the form of the "current potential" is defined by the non-local expression we have demonstrated that such a dualization of the Yang-Mills requires non-locality. But the latter does not spoil the general scheme of constructing duality-symmetric theories à la PST, though the trace of the non-locality can be observed in the gauge transformations of the doubled field YM action.

The same story happens in the gravity case [22]. Indeed, after resolving the torsion free constraint in the first order formulation of the Einstein-Hilbert action, one can present the gravity equation of motion in a similar to the eq. (4) form. Therefore in framework of the standard approach dualizing the gravity requires introducing non-localities too. However, we have mentioned above that there is an alternative way of non-abelian generalization of electric-magnetic duality based on the loop space formulation of a gauge theory in D=4 space-time dimensions [20]. An analog of such a formulation of gravitational theory is nothing but the Ashtekar-Sen approach (see e.g. [23], [24] for reviews). It would be interesting to figure out how the loop space approach could be reformulated to describe a duality-symmetric theory where the original and the dual potentials will appear on equal footing.

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